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Abstract

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Keywords

Exponential, Gamma and mixed Distribution, Survival function, Bayesian estimation, and Maximum likelihood Estimation.

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ABSTRACT

In this paper, we are used Bayesian on survival function estimator based on the mixed distribution of exponential distribution as primary distribution and Gama distribution as a function of probability of data, and the data was collected from Rizgari Hospital Erbil for stroke brain patients between 2015 and 2016. On the other hand, we compared with the traditional method that assumed exponential and Gamma distributions based on the Goodness of Fit tests depended, Since the value of the calculation χ^2 is equal to (10.767), It is less than the value of the tabular χ^2 which equals (11.345) for the variable x, that's mean that we accept the null hypothesis (H_0) which states that the data is distributed exponential distribution and this is confirmed by the P. value Which equals (0.014) Which is greater than the value of the moral level 1% We conclude that the data have an exponential distribution. While, for the variable t again distributed the Gamma distribution, because the statistical Cal. χ^2 is less than tab. χ^2 which are equal to (0.476, 11.335) respectively, that's means accepted the null hypotheses. We are also confirmed that the P. value equal to (0.924) is greater than level 1%. By using EasyFit Program, as well as using MATLAB and SPSS statistical programs. We concluded that the mixed and proposed combination of survival function for brain stroke was expectancy, appropriate and efficient.

H_0: The variables x is distributed exponential distribution.

H_1: The variables x is not.

H_0: The variables t is distributed Gamma distribution.

H_1: The variables x is not.

Keywords : *Exponential, Gamma and mixed Distribution, Survival function, Bayesian estimation, and Maximum likelihood Estimation.*

1. INTRODUCTION

Brain stroke is a most important healthiness burden in Iraq specific in Kurdistan as well as worldwide. Indeed stroke happen when blood provide to the brain is not working, the hungry brain of nutrients and oxygen. Brain cells start in on to die if blood run is stopped for more than a few seconds. While longer the supply is periodic, the more possible like there is to be permanent and incapacitating brain injure. A stroke is a health check urgent situation that requires urgent treatment (Soman et al.,, 2016). On the other hand the strokes have an effect on the brain depends on which element of the brain suffers harm. The intelligence stem controls your breathing, heartbeat, and blood pressure; controls your speech, swallowing, hearing, and eye movements. The pulses sent by other parts of the brain travel through the brainstem on their way to the various parts of the body. We depend on brain stem function to survive. Stem brain stem threatens the body's vital functions, making it a life-threatening condition. (Pietrangelo, 2016). In the United States, stroke is the third leading cause of death and an effective factor for severe disability, and die more than 140,000 people each (American Heart Association, 2009) year. It is expected that the prediction of stroke and early treatment will substantially increase the risk of stroke.

Data analysis and many medical studies have been carried out to classify effective career predictors. Framingham in Wolf et al. (1991) published a set of risk factors that may be related to stroke such as age, including age, systolic blood pressure, use of antihypertensive therapy, diabetes mellitus, smoking , Previous cardiovascular disease, an a trial test, and left ventricular hypertrophy by electrocardiogram, creatinine level, walking time 15 feet, and others (McGinn, et al., 2008).

2. METHODOLOGY

Let *t* denote a continuous non-negative random variable representing survival time, the probability density function for two parameter Gamma distributions has the following Gupta, R. D. and Kundu, D. (1999).

2-1 MAXIMUM lIKELIHOOD ESTIMATION FUNCTION (MLE)

The maximum probability, also called the maximum probability method, is to looking for one or more parameter values for a given statistic that increases the distribution of known probabilities. The maximum probability estimate for the parameter is determined. We assume there is a sample of $t_1,t_2,\ldots,t_n$ the n independent and evenly distributed observations coming from a distribution with an unknown probability density function $f(t_i)$, The $f(t_0)$ value is unknown and is called the true value of the parameter vector. It is desirable to find an estimate that is as close as possible to the true value α_0. The two variables can be either two or both. In order to the maximum likelihood method, the first one steps the joint density function for all observations. For a separate and evenly distributed sample, this density function is general (Lemke, D. 2016).

$$f(t_1,t_2,...,t_n/\alpha) = f(t_1/\alpha)\, f(t_2/\alpha)...f(t_n/\alpha) \qquad (1)$$

Now consider this function from another point of view by considering the observed values $t_1,t_2,\ldots,t_n$ as fixed "parameters" for this function, while, the function variable allowing a free change;

$$l(\alpha;t_1,t_2,...,t_n) = f(t_1,t_2,...,t_n/\alpha) = \prod_{i=1}^{n}(t_i/\alpha) \qquad (2)$$

Note that ";" refers to a separation between the two input argument classes: the parameters (α, $t_1,t_2,\ldots,t_n$) In practice, it is often more convenient to work with a natural logarithm for a probability function.

$$Ln\, l(\alpha;t_1,t_2,...,t_n) = \sum_{i=1}^{n} Ln\, f(t_i/\alpha) \qquad (3)$$

the finally formula we dependent on the following equation.

$$\hat{l} = \frac{1}{n}\ln l \qquad (4)$$

The maximum likelihood estimator agrees with the most likely annoying estimate if there is exponential posterior distribution on the parameters α. In fact, the maximum revaluation of the parameters that increases the probability of given data provided by the Bayesian theory is:

$$P(\alpha/t_1,t_2,...,t_n) = \frac{f(t_1,t_2,...,t_n/\alpha)\,P(\alpha)}{P(t_1,t_2,...,t_n)} \qquad (5)$$

Where; P(α) is the prior distribution for the parameter α

($t_1,t_2,\ldots,t_n$) is the probability of the data averaged over all parameters. Since the denominator is independent of α, and the Bayesian estimator is obtained by maximizing $f(t_1,t_2,\ldots,t_n/\alpha)\,P(\alpha)$ respect to α. If we further assume that the prior *P(α)* is a exponential distribution, the Bayesian estimator is obtained by maximizing the likelihood function Thus $f(t_1,t_2,\ldots,t_n\ /\alpha)$ the Bayesian estimator coincides with the maximum likelihood estimator for a uniform prior distribution *P(α)*.

2-2 SURVIVALDISTRUTION

Beginning with the simply definition for survival distribution on the possibility that the event there is no case of interest after the time *t*. If *T* specified time until death, so the survival function *S(t)* denoted probability of surviving further than time *t*. We start with probability density function which is below.

$$f(t,\alpha,\beta)=\frac{1}{\Gamma\alpha\beta^{\alpha}}t^{\alpha-1}\exp(-t/\beta)\qquad t>0\quad \alpha,\beta>0\qquad (6)$$

And the corresponding cumulative distribution function (CDF), survival and hazard functions as follows;

$$F(t;\alpha,\beta)=(1-\exp(-t\alpha)^{\beta};\qquad (7)$$

$$S(t;\alpha,\beta)=1-F(t;\alpha,\beta)=\exp(-t\alpha)^{\beta}\qquad (8)$$

$$h(t;\alpha,\beta)=\frac{f(t;\alpha,\beta)}{S(t;\alpha,\beta)}\qquad (9)$$

Where *α,β* are the shape and location parameters respectively, on the other hand the probability density, cumulative distribution, survival and hazard functions for the exponential distribution are as following Raqab and Ahsanllah (2001) and Raqab (2002):

$$f(t;\lambda)=\lambda\exp(-\lambda t)\qquad \lambda>0\qquad (10)$$

$$F(t;\lambda)=1-\exp(-\lambda t)\qquad (11)$$

$$S(t;\lambda)=\exp(-\lambda t)\qquad (12)$$

$$h(t;\lambda)=\lambda\qquad \text{constant hazard funcation}\qquad (13)$$

2-3 BAYESIAN ESTIMATION (THE PRIOR AND POSTERIOR DISTRBUTION)

A normal select for the prior of parameters $\alpha, \beta,$ and λ could be assume that the three parameters are independent and that, α ~G(a_0,b_0), β ~G(a_1,b_1) where G(a,b) denotes the gamma distribution with mean equal to $\alpha\beta$ and variance $\alpha\beta^2$, while the parameter λ is truncated positive exponential distribution Kotz, et.al(2003). All parameter above are chosen to reflect prior knowledge. The life time of a system with one component has the probability density function (PDF), for exponential and gamma distribution where $\alpha, \beta,$ and λ are known then:

We can assume $\alpha, \beta,$ by using maximum likelihood estimation as follows;

$$\hat{\alpha} = \frac{\bar{x}^2}{S^2} \text{ and } \hat{\beta} = \frac{S^2}{\bar{x}} \qquad (14)$$

$$f(t \mid \lambda) = \lambda \exp(-\lambda x) \qquad (15)$$

$$g(\lambda \mid \alpha, \beta) = \frac{1}{\Gamma\alpha \beta^{\alpha}} \lambda^{\alpha-1} \exp(-\lambda / \beta) \qquad \lambda > 0 \quad \alpha, \beta > 0 \qquad (16)$$

Where *t* is time of life patients for 2015 in equation (15), but *t* in formula (16) we can depend on the data 2016. However, to find the posterior distribution for above formula we have;

$$v(\lambda, \alpha, \beta \mid x) = \frac{f(x \mid \lambda) * g(\lambda \mid \alpha, \beta)}{\int_{\lambda} f(x \mid \lambda) * g(\lambda \mid \alpha, \beta) d\lambda} \qquad (17)$$

Dependent on the equation (17) we could work some mathematical on it, until get the posterior distribution;

$$v(\lambda, \alpha, \beta \mid t) = \frac{\lambda \exp(-\lambda x) * \frac{1}{\Gamma\alpha \beta^{\alpha}} t^{\alpha-1} \exp(-\lambda / \beta)}{\int_{t} \lambda \exp(-\lambda x) * \frac{1}{\Gamma\alpha \beta^{\alpha}} t^{\alpha-1} \exp(-\lambda / \beta) d\lambda} \qquad (18)$$

Rearranging and some mathematical work on the above equation, we get equation (19), below:

$$v(\lambda, \alpha, \beta \mid t) = \frac{(x + \beta^{-1})^{\alpha+1}}{\Gamma(\alpha+1)} \lambda^{\alpha} \exp(-\lambda(x + \beta^{-1})) \qquad (19)$$

Where $v(\lambda, \alpha, \beta \mid x)$ ~Gamma $(\alpha + 1, (x + \beta^{-1}))$ (20)

Furthermore, the survival function for the posterior distribution, depend on the probability density function for exponential is as follows:

$$S(t,\lambda) = p(x > t) = \int_t^\infty \lambda \exp(-\lambda x)dx \qquad (21)$$

Integrating the survival function in equation (21), we get;

$$S(t,\lambda) = \exp(-\lambda t) \qquad (22)$$

To find the Bayesian estimator survival function depends on the real survival function we have;

$$\hat{S}(t,\lambda,\alpha,\beta) = E[S(t)\,|\,x] = \int_0^\infty S(t,\lambda) * v(\lambda,\alpha,\beta\,|\,x)d\lambda \ \ \text{since}\,\lambda > 0 \qquad (23)$$

Again some mathematical work on above equation (23), then we gets equation as follow:

$$\hat{S}(t;\alpha,\beta) = [\frac{x+\beta^{-1}}{t+x+\beta^{-1}}]^{\alpha+1} \qquad (24)$$

Thus, depend on the above equation, we can assume that cumulative density, probability density functions and hazard function for Bayesian estimator as follow;

$$\hat{f}(t;\alpha,\beta) = \hat{F}'(t;\alpha,\beta) = -\hat{S}'(t;\alpha,\beta) \qquad (25)$$

After the derivatives and some mathematical work from equation (24) we can find the probability density function as follow:

$$\hat{f}(t;\alpha,\beta) = \frac{(x+\beta^{-1})^{\alpha+1}(1+x+\beta^{-1})(\alpha+1)}{(t+x+\beta^{-1})^{\alpha+2}} \qquad (26)$$

So, the hazard function is;

$$\hat{h}(t;\alpha,\beta) = \frac{(1+x+\beta^{-1})(\alpha+1)}{(t+x+\beta^{-1})(x+\beta^{-1})^{\alpha}} \qquad (27)$$

Where α,β and t are known depend on the data 2015, and also x we assume on the data 2016.

3. TECHNIQUES AND PATIENTS DATA

The data covers the 58 patients, 24 female and 34 male from January 2015 and 2016 to December 2015 and 2016 and the data was collected from Rzgary Teaching Hospital in Hawler city-Iraq. After we are organized the data and also we analyzed by using the Easyfit 5.6 standard Program.

3.1 STATISTICAL ANALYSES

This paper considers apply on the secondary data based on laboratory investigations. These data are supplied by Rizgary Teaching Hospital, Erbil, Iraq for the year 2015 and 2016. The data collected from existing databases in terms of the survival time of death which are shows in (Appendix A & B). The data includes the factors affecting the brain stroke which are age, survival time, time inter and exit patient on hospital. The data translated into codes using a particularly designed coding sheet, after that converted to computerized database. The specific statistical advice was required and EasyFit 5.6 standard program used. First all after the estimate parameter for the gamma and exponential then mixed together, and find baysen estimation which are prior and posterior expectations, secondly compare between classically and new model.

3.2 RESULTS

To determine the survival function for the gamma, exponential and mixed distributions on the brain stroke patients in the Rizgary Teaching Hospital. This is performed by finding the survival function curve for the selected time of death, beginning with first case depend on the 29 observation, to estimate scale parameter for the exponential distribution $\hat{\lambda}$ which is equal to (0.046), and the P- value of Chi-square and Kolmogrov simirnov are equal to (0.014, 0.06), which is grater than (0.010), while the value of the mean, variance and standard division are equal to (21.759, 288.260 and 16.978) respectively. To estimate the parameters location and scale for the gamma distribution are equal to $\hat{\alpha}$ =1.9278, $\hat{\beta}$ =10.607, and the P- value of Chi-square and kolmogrov simirnov are (0.924, 0.954), again which is greater than the level value (0.01), which is statistical significant, although the value of mean is (20.448), variance (216.900) and standard division (14.727). Know we are going to the second case which is mixed exponential and gamma distributions, the observation for these state is 58. Moreover, the value of parameters $\hat{\alpha}$ and $\hat{\beta}$ are (1.792, 11.779) respectively, and the P- values of Chi-square is equal to (0.226), and kolmogrov simirov is (0.443), also it is statistical significant, which is greater than that significant level (0.01). Moreover, the value of the mean, variance and standard division are equal to (21.103, 248.590 and 15.767) respectively. The comparison of the classically result and new model are the same consequences and the Figures support states which are shown from 1 to 12.

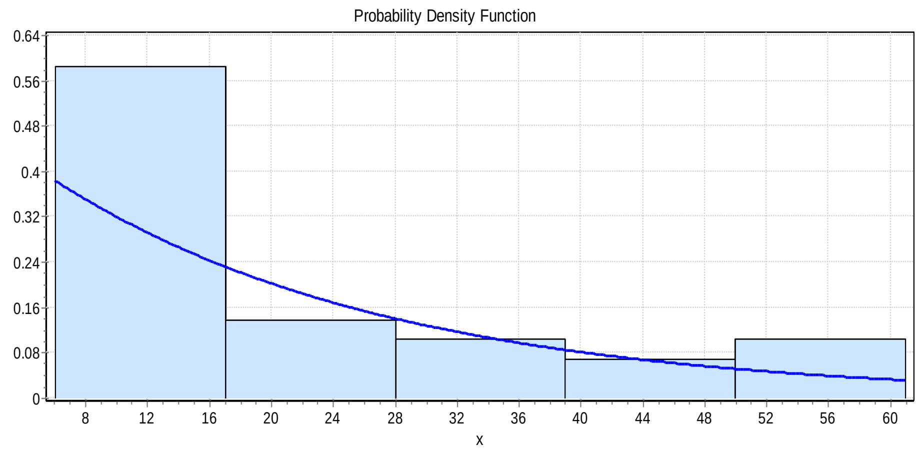

Figure 1. Probability density function curves for the exponential distribution (x)

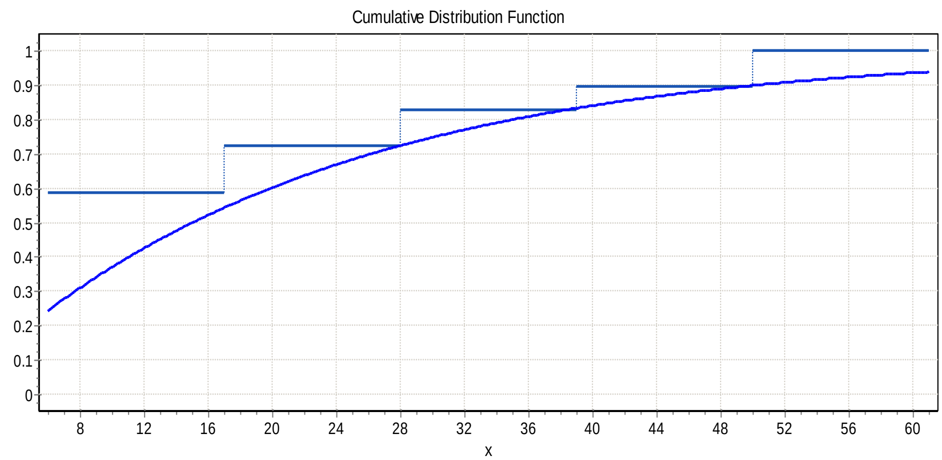

Figure 2. Cumulative distribution function curves for the exponential distribution (x)

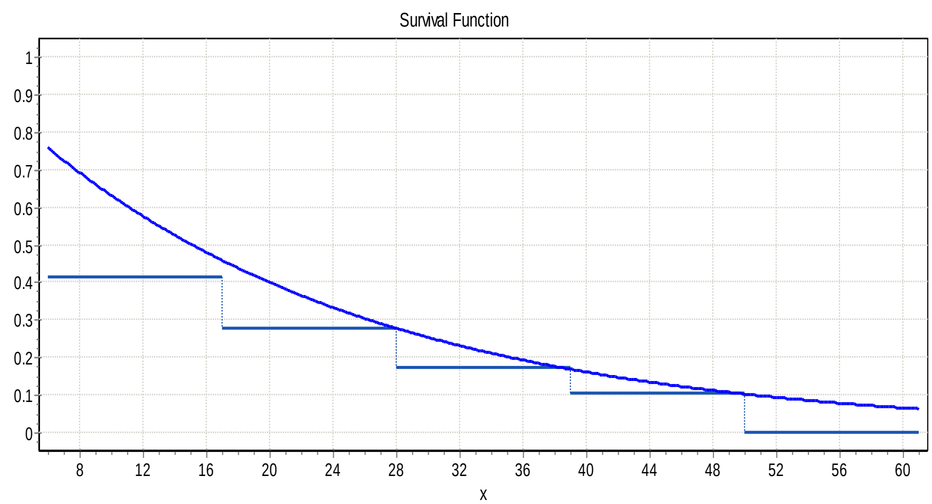

Figure 3. Survival function curves for the exponential distribution (x)

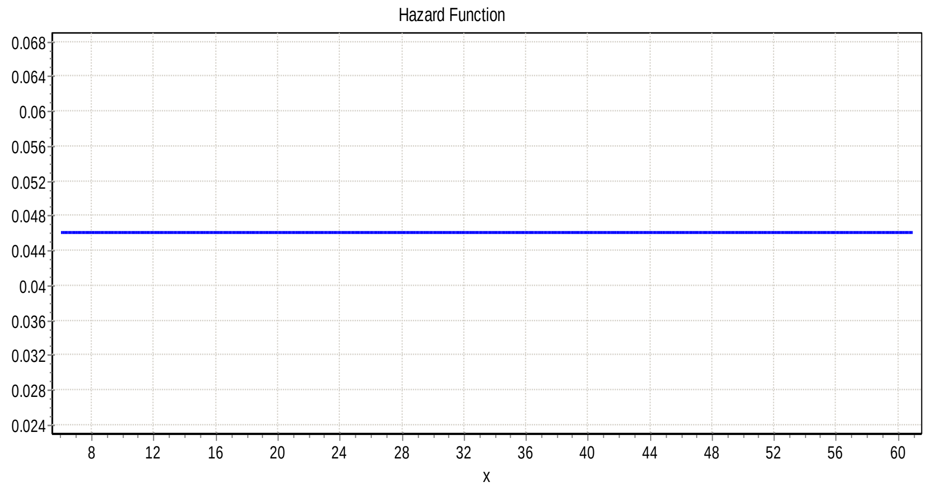

Figure 4. Hazard function curves for the exponential distribution (x)

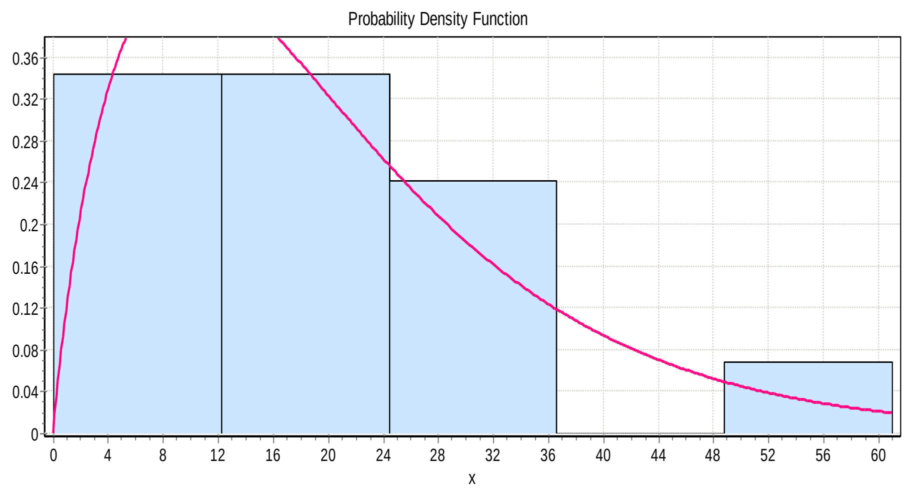

Figure 5. Probability density function curves for the gamma distribution (t)

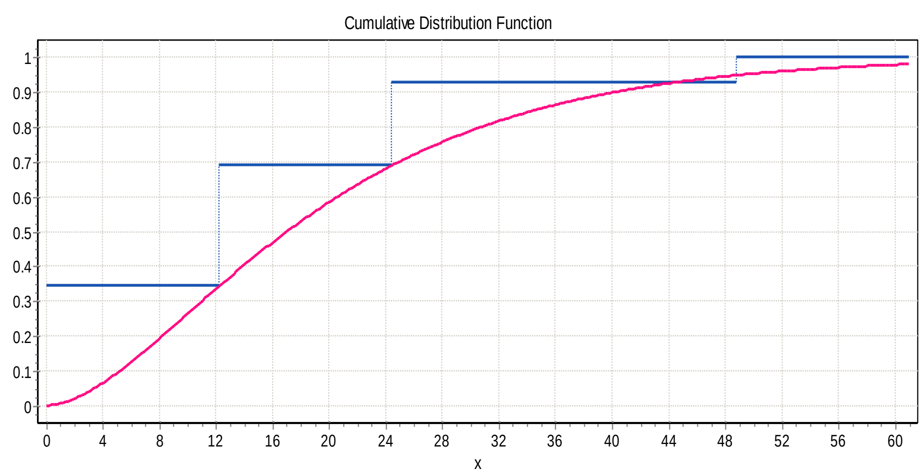

Figure 6. Cumulative distribution function curves for the gamma distribution (t)

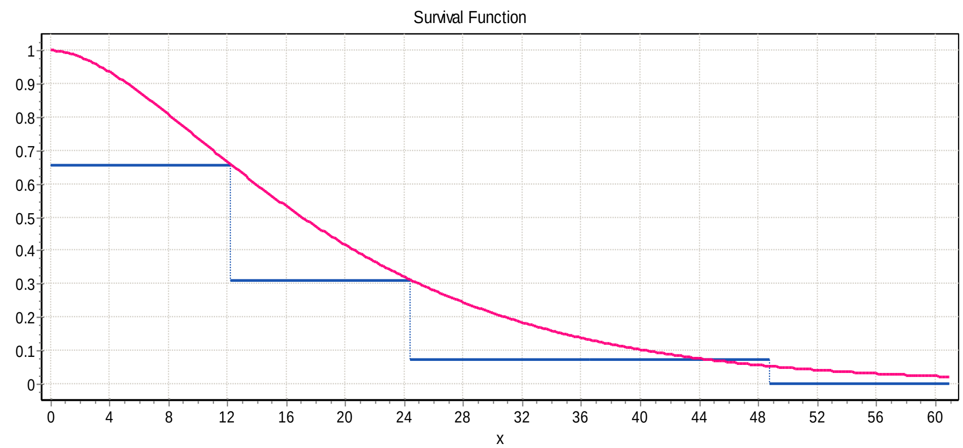

Figure 7. Survival function curves for the gamma distribution (t)

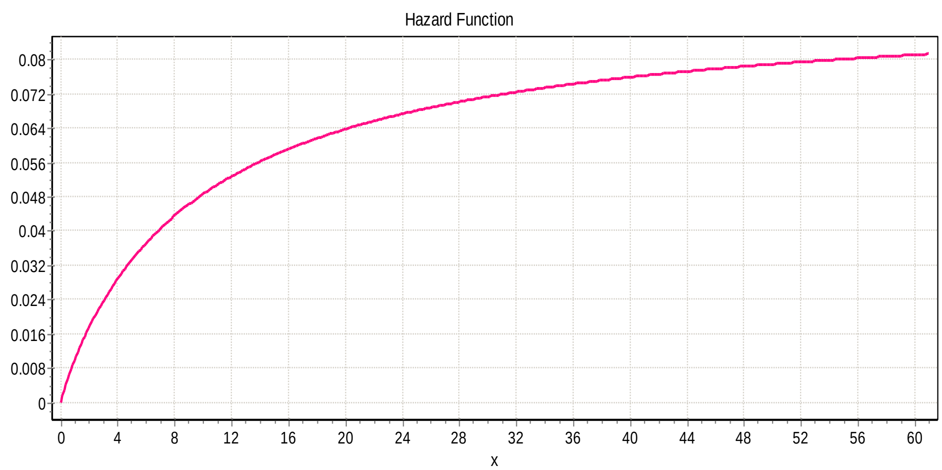

Figure 8. Hazard function curves for the gamma distribution (t)

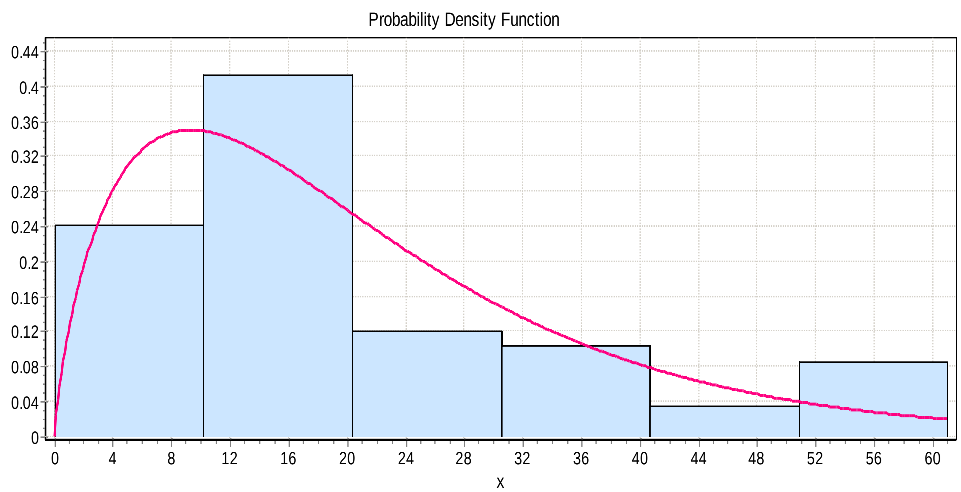

Figure 9. Probability density function curves for the mixed distribution (x,t)

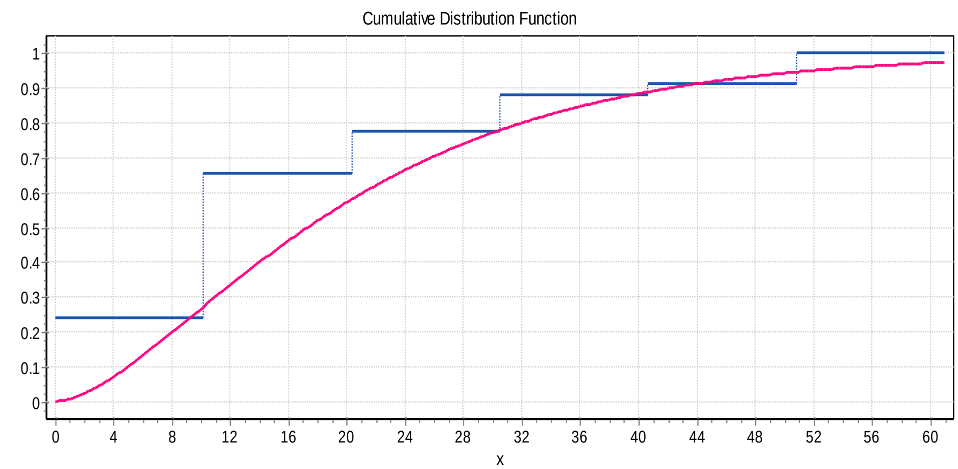

Figure 10. Cumulative distribution function curves for the mixed distribution (x,t)

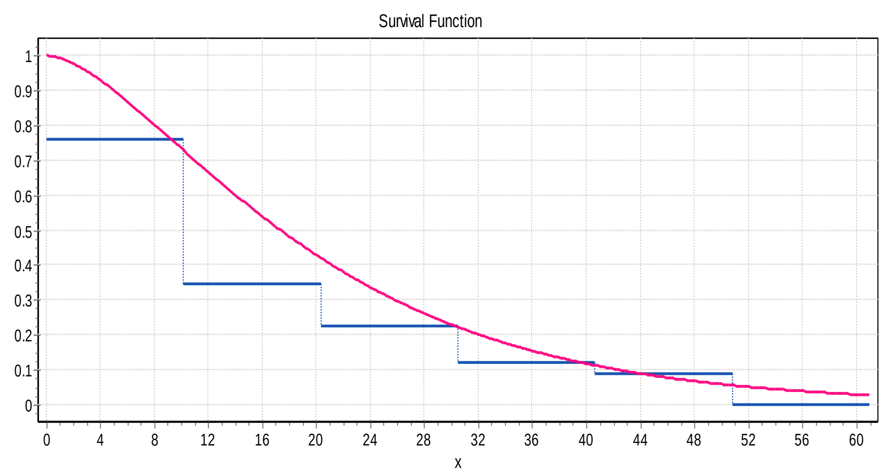

Figure 11. Survival function curves for the mixed distribution (x,t)

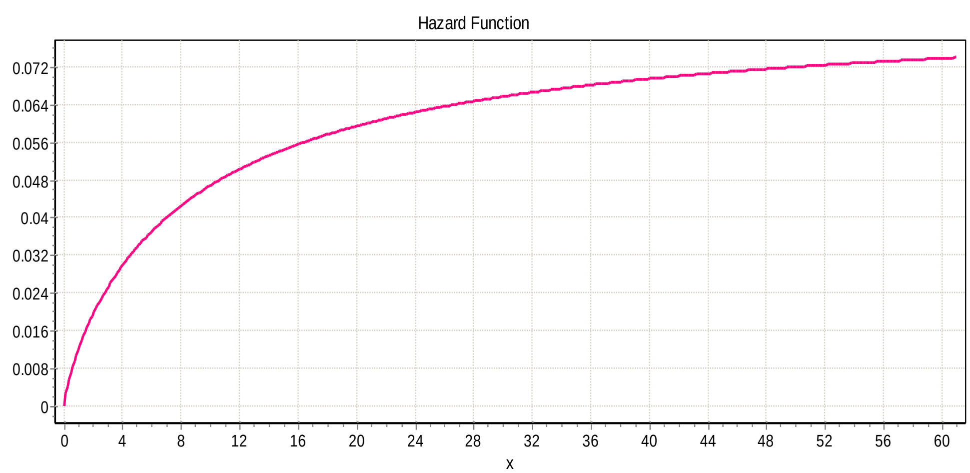

Figure 12. Hazard function curves for the mixed distribution (x,t)

4. DISCUSSION AND CONCLUIONS

They have been used in a large-scale analysis of survival in public health research, physicians and epidemiological research in health, social and behavioral economics studies. The relationship between a particular event and "potential risk factors", Or estimating the causal or predictive model is often of interested. The survival analysis is most appropriate for longitudinal studies where the event of interest, when it happens, however, it is frequently used with display data. We construed on the Bayesian estimator with survival function, the first case to analyze survival time data. Using maximum likelihood method to estimate exponential parameter, gamma distributions and using Bayesian method to estimate the parameters of both combined distributions. Although, the survival function for the gamma was very close to zero then that exponential. However, the survival function for the mixed distribution it's also approach to zero, which mean that they model was very well. On the other hand, depended on the value of Kolmogrov- simirnov for exponential distribution is (0.241), for gamma is equal to (0.107) and for the mixed is (0.110), which were statistically significant. Again, depending on our case study, incidence of brain stroke for the year 2015 and 2016 in the Rizgary Teaching Hospital, Erbil - Kurdistan Region of Iraq. On the table appendix B and C, the survival for the patients from 2016 is less than in 2015, the survival function carve in 2015 from zero until 14 days decrease which are equal to (1.00 - 0.133), but in 2016 from 6 and 13 days is the same survival function in 2015. However, the hazard function from 2015 and 2016 start for the first time until 14 increase routinely (0.011-0.105). Although, in the second around time which are from 16 and 30 days, the survival function 2015 and 2016 are equal to (0.119 – 0.079). The hazard function for the same around time are equal to (0.004 -0.009). We recommended that, firstly depending on the Bayesian on survival and hazard functions in the analysis of brain stroke diseases, and to conduct other research and studies to estimate the Bayesian on survival and hazard function based on other statistical distributions (goodness of fit), that are more suitable for data according to the kolmogrov-simirov and chi-square test differences. Secondly, the survival and failure function could be found for continuous distributions other than the exponential and Gama distributions, especially those continuous distributions that have to do with life tests, survival time and failure. These steps can also be applied to all malignancy diseases that affect human beings because of their high value, especially those diseases that cause death in many cases, including cancer, in addition to finding the parameters of survival and risk functions.

the following table showing the test of kolmogrov-simirov and chi-square for the exponential, gamma and mixed distributions.

Table1:Kolmogrov-simirov and Chi-square tests for the exponential, gamma and mixed distributions

Distributions	Test	Statistical Calculation	Tabulation Calculation	Significant Value (P. value)
Exponential Distribution	Kolmogrov	0.241	0.246	0.06
	Chi-square	10.767	11.345	0.014
Gamma Distribution	Kolmogrov	0.091	0.295	0.954
	Chi-square	0.476	11.332	0.924
Mixed Distribution	Kolmogrov	0.111	0.175	0.443
	Chi-square	6.923	11.070	0.226

الملخص

في هذا البحث، استخدمنا بيزيان على مقدر وظيفة البقاء على قيد الحياة على أساس التوزيع المختلط للتوزيع الأسي كتوزيع أولي وتوزيع كاما كدالة لاحتمال البيانات، وتم جمع البيانات من مستشفى رزكاري اربيل لمرضى الدماغ السكتة الدماغية بين عامي 2015 و2016 ومن ناحية أخرى، قارنا مع الطريقة التقليدية التي افترضت توزيعات الأسية و كاما على أساس اختبارات حسن المطابقة معتمدة، وبما أن قيمة الحساب كا- سكوير تساوي (10.767)، وهي أقل من قيمة الجدولية التي يساوي (11.345) للمتغير توزيع الاسي (x) وهذا يعني أننا نقبل الفرضية العدم (H0) التي تنص على أن البيانات يتوزع توزع التوزيع الأسي وهذا ما تؤكده قيمة (P-value) التي تساوي (0.014) وهي أكبر من قيمة المستوى المعنوي 1٪ نستنتج أن البيانات لها توزيع أسي. بينما، لمتغير توزيع كاما (t) مرة أخرى، لأن اختبار كا-سكوير الإحصائي أقل من قيمة الجدولية التي تساوي (0.476، 11.335) على التوالي، وهذا يعني قبول الفرضيات العدم. ويؤكد أيضا أن قيمة (P-value) يساوي (0.924) أكبر من مستوى المعنوي 1٪. باستخدام برنامج EasyFit فضلا عن استخدام البرامج الإحصائية (SPSS, Matlab). ونستنتج من ذلك إلى أن مزيج مختلطة والمقترحة من وظيفة البقاء على قيد الحياة لسكتة الدماغ كان متوقعا، مناسبة وفعالة.

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Appendix A: General table for the survival function for patient's (Censored and Death) years 2015 &2016

No.	Time2015&16	Survival Time	Status	No.	Time2015&16	Survival Time	Status	No	Time2015&16	Survival Time	Status
1	1	8	1	34	2	30	0	67	2	11	0
2	1	29	1	35	2	13	1	68	2	42	1
3	1	33	1	36	2	8	0	69	2	14	1
4	1	17	1	37	2	21	1	70	2	12	1
5	1	61	1	38	2	3	0	71	2	61	1
6	1	0	1	39	2	17	1	72	2	12	0
7	1	30	1	40	2	17	0	73	2	24	0
8	1	35	1	41	2	31	0	74	2	26	0
9	1	31	1	42	2	62	0	75	2	17	1
10	1	61	1	43	2	10	1	76	2	8	1
11	1	31	0	44	2	12	1	77	2	15	1
12	1	3	0	45	2	11	1	78	2	6	1
13	1	32	1	46	2	16	1	79	2	10	1
14	1	2	0	47	2	20	1	80	2	13	1
15	1	5	1	48	2	14	0	81	2	11	0
16	1	31	0	49	2	16	0	82	2	44	1
17	1	92	0	50	2	11	1	83	2	11	0
18	1	31	0	51	2	13	1	84	2	13	1
19	1	31	0	52	2	4	1	85	2	6	1
20	1	32	0	53	2	8	1	86	2	9	1
21	1	33	0	54	2	10	1	87	2	36	1
22	1	25	0	55	2	14	1	88	2	24	0
23	1	34	0	56	2	21	1	89	2	28	0
24	1	30	1	57	2	61	1	90	2	20	1
25	1	35	0	58	2	61	1	91	2	13	1
26	1	12	0	59	2	15	1	92	2	18	1
27	1	23	1	60	2	8	1	93	2	34	1
28	2	30	0	61	2	5	0	94	2	6	0
29	2	31	0	62	2	61	0	95	2	9	1
30	2	15	0	63	2	61	0	96	2	12	1
31	2	4	0	64	2	12	1	97	2	33	0
32	2	13	0	65	2	7	0	98	2	25	0
33	2	18	1	66	2	12	1	99	2	19	0
								100	2	29	1

Appendix B: Specific table for the survival function for patient's (Censored) years 2015 &2016

No.	Time Day(t)	Status	Time Day(x)	Status	variable (t)	variable(x)	(x+(1/B))	t+x+(1/B))	S(t)=[(x+(1/B))/t+x+(1/B))]^α+1

1	8	1	18	1	0	6	6.0943	6.0943	1.0000
2	29	1	21	1	4	6	6.0943	10.0943	0.2282
3	33	1	10	1	5	8	8.0943	13.0943	0.2446
4	17	1	11	1	8	8	8.0943	16.0943	0.1337
5	61	1	13	1	8	9	9.0943	17.0943	0.1576
6	0	1	8	1	10	9	9.0943	19.0943	0.1140
7	30	1	10	1	10	10	10.0943	20.0943	0.1332
8	35	1	14	1	11	12	12.0943	23.0943	0.1505
9	31	1	21	1	11	12	12.0943	23.0943	0.1505
10	61	1	15	1	12	12	12.0943	24.0943	0.1329
11	32	1	8	1	13	12	12.0943	25.0943	0.1180
12	5	1	12	1	13	13	13.0943	26.0943	0.1328
13	30	1	12	1	14	13	13.0943	27.0943	0.1190
14	23	1	42	1	16	13	13.0943	29.0943	0.0966
15	18	1	14	1	17	14	14.0943	31.0943	0.0986
16	13	1	12	1	17	15	15.0943	32.0943	0.1099
17	21	1	61	1	18	15	15.0943	33.0943	0.1004
18	17	1	17	1	20	17	17.0943	37.0943	0.1035
19	10	1	8	1	21	18	18.0943	39.0943	0.1048
20	12	1	13	1	23	20	20.0943	43.0943	0.1071
21	11	1	13	1	29	21	21.0943	50.0943	0.0795
22	16	1	9	1	30	29	29.0943	59.0943	0.1256
23	20	1	36	1	30	34	34.0943	64.0943	0.1575
24	11	1	20	1	31	36	36.0943	67.0943	0.1628
25	13	1	13	1	32	42	42.0943	74.0943	0.1910
26	4	1	12	1	33	44	44.0943	77.0943	0.1948
27	8	1	29	1	35	61	61.0943	96.0943	0.2655
28	10	1	61	1	61	61	61.0943	122.0943	0.1317
29	14	1	61	1	61	61	61.0943	122.0943	0.1317

Appendix C: Specific table for the hazrd function for patient's (Censored) years 2015 &2016

No.	Time Day (t)	Status	Time Day(x)	Status	variable (t)	variable (x)	(1+x+1/ B)	(1+x+1/ B)(α+1)	(t+x+1/B)	(x+1/B)^α	(t+x+1/B)(x +1/B)^α	h(t)=(1+x+ 1/B)(α+1)/(t+x+1/B)(x +1/B)^α
1	8	1	18	1	0	6	7.094	20.771	6.094	32.597	198.655	0.105
2	29	1	21	1	4	6	7.094	20.771	10.094	32.597	329.043	0.063
3	33	1	10	1	5	8	9.094	26.626	13.094	56.336	737.684	0.036
4	17	1	11	1	8	8	9.094	26.626	16.094	56.336	906.693	0.029
5	61	1	13	1	8	9	10.094	29.554	17.094	70.521	1205.499	0.025
6	0	1	8	1	10	9	10.094	29.554	19.094	70.521	1346.540	0.022
7	30	1	10	1	10	10	11.094	32.482	20.094	86.230	1732.730	0.019
8	35	1	14	1	11	12	13.094	38.337	23.094	122.180	2821.655	0.014
9	31	1	21	1	11	12	13.094	38.337	23.094	122.180	2821.655	0.014
10	61	1	15	1	12	12	13.094	38.337	24.094	122.180	2943.835	0.013
11	32	1	8	1	13	12	13.094	38.337	25.094	122.180	3066.015	0.013
12	5	1	12	1	13	13	14.094	41.265	26.094	142.400	3715.839	0.011
13	30	1	12	1	14	13	14.094	41.265	27.094	142.400	3858.239	0.011
14	23	1	42	1	16	13	14.094	41.265	29.094	142.400	4143.040	0.010
15	18	1	14	1	17	14	15.094	44.193	31.094	164.107	5102.780	0.009
16	13	1	12	1	17	15	16.094	47.121	32.094	187.290	6010.956	0.008
17	21	1	61	1	18	15	16.094	47.121	33.094	187.290	6198.247	0.008
18	17	1	17	1	20	17	18.094	52.976	37.094	238.062	8830.757	0.006
19	10	1	8	1	21	18	19.094	55.904	39.094	265.637	10384.904	0.005
20	12	1	13	1	23	20	21.094	61.760	43.094	325.135	14011.471	0.004
21	11	1	13	1	29	21	22.094	64.688	50.094	357.047	17886.025	0.004
22	16	1	9	1	30	29	30.094	88.110	59.094	663.634	39217.011	0.002
23	20	1	36	1	30	34	35.094	102.74	64.094	900.957	57746.191	0.002
24	11	1	20	1	31	36	37.094	108.60	67.094	1005.61	67470.792	0.002
25	13	1	13	1	32	42	43.094	126.17	74.094	1352.62	100221.86	0.001
26	4	1	12	1	33	44	45.094	132.02	77.094	1479.24	114041.45	0.001
27	8	1	29	1	35	61	62.094	181.80	96.094	2773.65	266532.24	0.001
28	10	1	61	1	61	61	62.094	181.80	122.09	2773.65	338647.22	0.001
29	14	1	61	1	61	61	62.094	181.80	122.09	2773.65	338647.22	0.001